Modeling the Characteristics of System of Systems

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Abstract – The term system of systems (SoS) has become a quite common expression with no single accepted definition. Based on analysis [1], five characteristics have been identified from an assortment of SoS descriptions. These characteristics are named autonomy, belonging, connectivity, diversity, and emergence, whose degrees of strength determines the foundation of any SoS. To utilize these characteristics, concrete definitions are presented in context and then applied to the development of a theoretical model. The resulting model forms the basis of a generic SoS instantiated as an agent-based modeling simulation. This simulation demonstrates the appropriateness of the characteristics and can be used to explore further aspects of a SoS. Modeling and experimenting on these characteristics will contribute to the field of systems engineering by providing a means to increase our understanding of SoS.

Keywords: Systems, system of systems, autonomy, belonging, connectivity, modeling, agent-based model

1 Introduction

System of systems (SoS) has become a readily accepted term to classify an arrangement of independent and interdependent systems that delivers unique capabilities, but is it something new or something more than just a complex traditional system? At first glance it would appear that a SoS is a type of multi-system based on the name alone, but the term system is also ambiguous. There have been many attempts at defining and depicting these complex systems based on experience, and while many of the definitions designate SoS as a new entity, there has been little convergence on a single definition.

In a review of the literature, Gorod, et al. [2] revealed many endeavors to define and characterize SoS and concluded that a characterization is a more optimal approach to understanding them. Given the diverse descriptions of SoS their characterizations are usually a list of various features that appear mostly anecdotal. While there is no consensus on what these characteristics may be, there has been some convergence. One of the more comprehensive studies to describe a set of characteristics was performed by Boardman and Sauser based on a cross reference of over 40 definitions. This study presented five SoS characteristics that distinguish a system from a SoS based on degrees of fluctuation. These characteristics are named autonomy, belonging, connectivity, diversity, and emergence [1], [2], [3], [4], [5], [6], [7]. These attributes were extracted from 41 different descriptions of SoS, but the given names are somewhat nebulous. Of these five characteristics, the first three of autonomy, belonging, and connectivity are identified as the competency set of the group, while diversity and emergence exploit the initial three. Tracing each competency set characteristic to the various descriptions elucidates their intended meanings.

Therefore, rather than performing merely another observational or qualitative study, we are proposing to model these characteristics and their properties to serve as the basis for our theoretical model of an ideal SoS, and then use a computer simulation to demonstrate this model. The next section presents definitions for the various terms needed in this study. In section 3, the SoS characteristics translate via the definitions into a theoretical model of a SoS, which is expressed mathematically. The mathematical description forms the basis of an agent-based model. Implementing the theory in a simulation demonstrates the appropriateness of the characteristics. As discussed in the conclusion of section 4, modeling a SoS should contribute to the field of systems engineering by investigating the impacts of certain decisions on a SoS as well as increasing our understanding of SoS in general.

2 System Terms

“What is the essence of a system and how can this be leveraged to explain the fundamental distinction between a system that is truly a SoS and one that is ‘merely’ a system (of parts)” [6]? To address this question, this section explores the various terms and adopts a set of definitions.

2.1 Traditional System

The word system is ubiquitous in everyday usage, but exactly what is meant by the term in the context of SoS? The basic concept can be found as far back as the time of Plato, but it has evolved and changed over the years [8]. A system as a whole is said to be more than the sum of its
A system can refer “to a group or complex of parts (such as people, machines, etc.) interrelated in their actions towards some goals” [11].

A similar definition, “A system is a set of interrelated components working together toward some common objective or purpose” [12].

We conclude a system possesses at least three qualities. First, it is composed of a set of elements with actions. Second, these elements are interrelated. Finally, the combination of components must have a common goal or purpose. Therefore the accepted definition of system for the purposes of this paper is a set of elements acting and interacting to achieve some common goal(s).

2.2 System of Systems (SoS)

There is no accepted definition of a SoS [13]. Among the many contributions, Kotov [14] describes a SoS as a compilation of distributed, complex component systems. Crossley [15] describes it as multiple, independent systems that interact for the purpose of a global goal. An analogous definition uses autonomous and diverse component systems [16]. Carlock and Fenton [17] state a SoS evolves slowly over time and is more complex than developing stand-alone systems. Heterogeneous systems within a SoS are integrated for the purpose of working effectively together. According to Kang and Mavris [18], the union of different individual systems forms a new SoS, which has a different function than any one of the individual systems. Lane and Boehm [19] agree that each individual system has its own purpose beside the SoS. Maier calls “collaboratively integrated systems” a SoS when the system possesses operational independence and managerial independence of its components and optionally geographic distribution, emergent behavior, and evolutionary development [13], [20]. Monarch and Wessel [21] substitute Maier’s independence with “operational autonomy” and “managerial autonomy.”

Shenhar et al. use a description of an Array, otherwise called a system of systems, as a distributed or large “collection of systems functioning together to achieve a common purpose” [22], [23]. To differentiate an Array, an Assembly or subsystem performs a function within a larger system, and a System is composed of Assemblies or parts [22]. Gorod, et al. was also able to show how Shenhar’s classification of Array, System, and Assembly could be described using Boardman and Sauser’s five characteristics based on a Flexibility Dynamic [7].

At an elementary level, there are three points common among these different definitions. First, a SoS is a system itself. Second, the systems that compose a SoS are systems, as their name indicates. Lastly, the constituent systems maintain a level of independence before and after joining a SoS [24]. According to Shah, Rhodes and Hastings, it is this last point that distinguishes a SoS.

Although as stated earlier, it is a characterization of SoS that we believe will allow us to reach deeper insights into what they are and how we may understand them. Thus, we refer back to the work by Boardman and Sauser.

The various descriptions from the literature provide ample evidence for these characteristics. Accordingly, this paper adopts the definition of a SoS as a type of system composed of traditional systems and distinguished by the dynamic properties of autonomy, belonging, connectivity, diversity, and emergence. The original rationales for all five characteristics are summarized in tables in [3]. We may argue that better terms are available, but the intentions of the terms are what matters.
If the term is confusing still, the provenance of this belonging is “shared mission” [1], [3]. Other sources of this characteristics include synergism [25] [26], interdependence [27], interoperability [26], effectively working together [17], interacting for the global mission [15], functioning together to achieve a common purpose [22] [23], and the contribution of the parts to the capabilities of the whole [27]. Compiling this information results in a definition for belonging within a SoS as the acceptance ability and need to make a valued contribution to the goal of the larger entity (e.g. a SoS). Depending on the system and the goal of the larger entity, the system’s contribution varies. Furthermore the SoS has to accept that the system’s contribution is sufficient. Since each system is unique, the contributions of diverse systems will impact the SoS goal differently.

2.5 Connectivity

Mathematical ecologist Richard Levins has declared that connectivity is “a fundamental property, an essential feature of the system” [28]. Arguably most of the SoS descriptions infer the connectivity of component systems. Thus connectivity appears to play a role in all systems.

Sauser, Boardman and Gorod [6] list connectivity as a trait of a SoS in that “the internal connectivity of the SoS is not presciently designed but emerges as a property of present interactions among holons.” Conversely for systems, the connections are mostly static.

Evidence of the characteristic is found in descriptions that the systems in a SoS are geographically distributed [13] [20], distributed [14], and connected [27]. In addition, most if not all of the descriptions imply connectivity of the systems in some form. Within a SoS, these connections are dynamic or possess evolutionary development [13] [20], self-organizing and adaptive [25]. Furthermore self-assembling systems make connections as needed. Consequently connectivity is defined as the capability to form connections as needed to benefit the entity.

2.6 Diversity and Emergence

The competency set is defined as the characteristics of autonomy, belonging, and connectivity. These three characteristics are fundamental to a SoS while diversity and emergence represent higher-order attributes derived in part using the competency set [4]. Therefore any valid study of the competency set will inherently include diversity and emergence since their presence is unavoidable.

For completeness, definitions are needed for diversity and emergence, and the definitions provided by Boardman and Sauser will be accepted. “A SoS should, out of necessity, be incredibly diverse in its capability as a system compared to the rather limited functionality of a constituent system, limited by design” [3]. The difference between autonomy and diversity is that diversity ensures multiple system capabilities while autonomy ensures these capabilities can complete the system goal. Additionally, “in a system, emergence is deliberately and intentionally designed in… Instead, a SoS has emergent capability designed into it by virtue of the other factors: preservation of constituent systems autonomy, choosing to belong, enriched connectivity, and commitment to diversity of SoS manifestations and behavior” [3].

Equipped with these definitions, we are ready to instantiate these terms not as a qualitative case study (as has been done to this point) but through modeling and simulation. A more concrete representation of the characteristics will demonstrate whether these attributes are capable of forming a SoS. If a model built from these characteristics fails to mimic a SoS, we will need to rethink our premises.

3 Modeling a SoS

At least as early as 1973, “there is a growing interest in abstract properties of complex systems as such and the ways of studying them …” [28]. Thus, rather than performing another observational study, we intend to use methods to model the essential characteristics of a SoS. We aspire that this model may serve as a paradigm of SoS by presenting an axiom set for potential SoS research. Moreover the resulting model forms the basis for a simulation of a generic SoS.

3.1 Theoretical Model

Systems engineering is a relatively young discipline [22] [23]. As such, any solid theoretical foundation is scarce compared to many of the other sciences. Compiling the definitions presented in the previous section, this paper proposes a theoretical model, developed mathematically. This model utilizes notation from basic set theory and first-order predicate logic.

As stated, a partial definition of a system is a set of elements. Therefore, represent any system as the set $S_i$. Then define its elements to complete the definition such that $A_i$ is the set of actions to achieve some common goal. Of course there are additional elements in $S_i$ such as the non-empty subset of goals, $G_i$ and possibly other undeclared elements, $E$, (Equation 1).

$$S_i = \{A_i, G_i, E_i\}, G_i \neq \emptyset, i \in \mathbb{Z}^+$$

Autonomy is defined as a system’s ability to complete system-level goals, which can only be accomplished by its set of actions, $A_i$. Consequently a system with more goals requires a larger set of actions to complete them. Therefore the autonomy of a system is directly proportional to the cardinality of its subset $A_i$ (Equation 2). In other words, the quantification of system actions is an element of autonomy. Of course autonomy is more than the number of actions in that it requires the
appropriate actions to achieve a goal. As a result the composition of $A_i$ is important albeit difficult to represent. Therefore the count is not necessarily a complete depiction but nonetheless an indicator of the characteristic. Similarly the cardinality of the set $G_i$ represents the number of goals for a system. The concept of diversity ensures that the set $A_i$ has multiple elements and there are multiple sets $A_i$ for different systems (Equation 3).

$$\text{Autonomy} \equiv |A_i|$$

$$\text{Diversity} \equiv A_i \neq \emptyset \land |A_i| > 1, i \in Z^* > 1$$

Since a SoS is a type of system, let $S^*$ represent the set corresponding to a SoS. This set has multiple component systems as its elements and a non-empty subset of goals, represented by $G^*$ (Equation 4). Noticeably absent from the definition are the actions found in a traditional system. Therefore the set $S^*$ does not overtly require a subset of actions.

$$S^* = \{S_{i1}, \ldots, S_{in}, G^*\}, n \in Z^* > 1, G^* \neq \emptyset$$

What does it mean for a system to contribute to the SoS goal? Given the scope of our definitions, it must imply the existence of some actions utilized in the achievement of the SoS goal. Although not explicitly stated, the component systems’ actions rectify the conspicuously missing SoS actions. Therefore belonging relates to the intersection of the system actions with the SoS goals, assuming a functional relation between actions and goals (Equation 5).

$$f: A_i \rightarrow G^* \mid (\forall g \in G^*) (\exists a \in A_i) (f(a) = g)$$

The intersection results in a subset of goal-actions, which does not create a very useful metric by itself. In order to standardize the belonging for each system, divide the cardinality of the intersection by the cardinality of the system’s actions. Another way to think about it, a system contributes a percentage of its actions to the purpose of the SoS goal. The resulting equation represents a measurement of belonging relating to the proportion of system effort contributed to the SoS. Note the divisor of the equation is the autonomy metric. Since the SoS accepts a system with a sufficient level of effort, there exists some threshold $\beta$ greater than zero (Equation 6).

$$\text{Belonging} \equiv \frac{|f(A_i) \cap G^*|}{|A_i|} \geq \beta_i > 0$$

Adhering to the accepted definition, belonging must be related to a system’s valued contribution, which is measured as the system’s advancement of the SoS goals. Let $g(i)$ be a particular system’s valued contribution to the SoS goal. Then its belonging threshold is inversely proportional to the contributed value (Equation 7).

$$\forall S_i (\exists \beta_i) (\beta_i^{-1} \propto g(i))$$

Next consider the characteristic of connectivity. If a system possesses the capability to connect to anything else, some of its undeclared elements must implement the connection. Let $C_i$ and $C_j$ be the subsets of static connectors for any two systems. In order for these two systems to connect, they must have a connector in common (Equation 8). No connection takes place when the two sets of connectors are disjoint. Furthermore, two connected systems will disconnect if either system contributes below the belonging threshold, regardless of the connectivity. As expected [6], each system has a static subset of connectors, but the set of connectors for the SoS is dynamic as systems join and leave the larger entity.

$$\text{Connectivity}_{i,j} = \begin{cases} \text{true} : & \exists k \{c_i, c_k\} \in C_i \times C_j \\ \text{false} : & \forall k \{c_i, c_k\} \notin C_i \times C_j \end{cases}$$

At first glance, the connectivity equation appears to only be a binary value of true or false. While this observation is correct for any pair of systems, dynamic connectivity emerges for a SoS from the complex nature of the multiple component systems. Therefore the range of possible connections has the potential to become countably infinite. Hence the ranges of values for all the equations adhere to expectations.

Finally, the existing set of factors gives rise to the characteristic of emergence. While missing from the mathematical descriptions, implementation of the model should inherently possess this trait.

### 3.2 Simulation Demonstration

Agent-based modeling (ABM) is one approach to implement this mathematical model via a computer simulation. ABM is a computational tool that can produce system functionality by programming specific elements of the system [29], [30]. A description of ABM states, “A system is modeled as a collection of autonomous decision-making entities called agents. Each agent individually assesses its situation and makes decisions on the basis of a set of rules” [30]. These agents are distinct parts of a software program and can interact based on each agent's rules [29]. Therefore ABM is an appropriate tool to explore the implementation of our SoS model. For this study we used AnyLogic as our ABM tool. AnyLogic is a modeling tool that supports common system simulation methodologies.

Therefore, when our mathematical model is executed within the AnyLogic environment we reveal instantiations that resemble a SoS. Our initial approach in this research was to develop a model that is generic for any SoS, and we therefore use a simple example to demonstrate what we have found. The chosen example emulates a collaboration of authors for a journal publication. The collaboration forms a SoS in that each author is an independent system yet the goal of publishing applies to the...
entire group. In the simulation, each agent represents one of three author-types with an assigned skill level. In addition, each agent has a randomly assigned goal set and randomly assigned set of connectors. A limited range of real numbers represents the goal set. Thus any two agents may have overlapping span of numbers. Although this technique is nonspecific, it represents a set of potential publication goals that may or may not coincide between authors. Next the skill level represents the maximum contribution available by an author. For convenience, the agents move around the screen to represent the chance encounter of authors. When two agents make contact, they have the option to connect. If they have any mutual connectors, they will connect, which simulates two authors deciding to collaborate. Alternately one could think of this connection as the ability for two authors to work together. Next the agents check to see if their goal sets overlap, in which case they form a SoS (see Figure 1). The contribution to the SoS goal depends on the percentage of overlapping goals. As new agents bump into the SoS, they have the option to connect also based on any mutual connectors. If they connect, the belonging value is calculated. If the belonging is less than a set threshold, the SoS rejects the new agent. Otherwise the new agent contributes a percentage of its skill to the SoS goal. Of course individual authors can work on their own papers, but this action does not help a SoS. Once a SoS publishes a journal paper, a new publication goal is randomly assigned by selecting a new range of numbers. Any agents failing to belong to this new goal leave the SoS. The remaining agents start working on this new goal, simulating a new journal article.

The resulting simulation demonstrates the mathematical model or at least mimics what we would expect from the SoS. This result encourages us to continue with this approach and verify the output is truly an acceptable SoS. While this ABM simulation is merely a demonstration, a more robust and proven ABM should offer additional insights into SoS.

4 Conclusions and Future Research

A system is defined as a set of elements acting and interacting to achieve some common goal. Composed of traditional systems, a SoS is distinguished by the degrees of strength for autonomy, belonging, connectivity, diversity, and emergence. Since diversity and emergence build from the competency set, the competency set should precede analysis of these subsequent characteristics. Using these characteristics and the basic definitions of system and SoS, a theoretical model was developed and described using set theory. As expected, the mathematical descriptions indicate a logical relationship among the characteristics and system.

The mathematical descriptions are implemented via computer simulation in order to demonstrate that they do resemble a SoS. Having achieved some initial success with the demonstration, the intention of future work is to develop the theoretical model further and implement it in a more robust agent-based model of a SoS. Initially the simulation will aid in the determination of empirical values appropriate for the unknown variables such as the belonging threshold and valued contributions. Once verifiable values have been found, we anticipate determining how altering the competency set characteristics impact the SoS, such as its resilience or efficiency. For example, what is the impact to a SoS by changing its goals? Another example examines the benefit of autonomy. While a greater level may benefit some SoS, excessive autonomy may cause a system to fail to belong. Future work will investigate these questions and others related to the impact of the characteristics. This understanding will advance the body of knowledge of systems engineering for SoS and may direct efforts for handling design and maintenance of SoS.

5 References


